Sell in May and Go Away:
Still Good Advice for Investors?

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Abstract

The "Sell in May and Go Away" (or Halloween) strategy continues to enjoy great popularity in practice. We explore whether this simple trading rule still offers an opportunity to earn abnormal returns. In contrast to prior studies, we consider sample periods during which adequate investment instruments were available for an effective implementation of the Halloween strategy. In addition, we account for when the first study confirming the Halloween effect was published in a top academic journal. To use the limited data in the most efficient way, and to avoid possible data-snooping biases, we implement a bootstrap simulation approach. We find that the Halloween effect strongly weakened or even diminished in recent years. Our results are robust across different countries and against various parameter variations. Overall, our findings support the theory of efficient capital markets.

Keywords: Sell in May, Halloween effect, bootstrap simulation, statistical inference, investment strategy

JEL classification: G11, G12, G14

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1. Introduction

The old, somewhat simplistic, investment strategy, "Sell in May and Go Away" (also known as the "Halloween" strategy), has enjoyed unbroken popularity in the academic literature as well as in actual investment practice. It posits that holding stocks from November through April, and then switching to cash from May through October, provides higher returns and lower risk than a conventional buy-and-hold strategy.

In their seminal study, Bouman and Jacobsen (2002) support the widespread hypothesis that a Halloween strategy offers opportunities for earning abnormal returns. Several more recent studies have confirmed that the effect is still alive and provides profitable investment opportunities (Jacobsen and Zhang, 2012; Andrade et al., 2013, Swinkels and van Vliet, 2012). However, if stock markets are informationally efficient, no such "anomaly" should exist over extended periods of time. As Fama (1970, 1991) and Jensen (1978) emphasize, in a semi-strong efficient market, it should be impossible to profit from publicly available information. And, if such risk-adjusted abnormal returns net of all costs are nevertheless possible, these investment opportunities should diminish quickly as the underlying strategy becomes more transparent.

In fact, it is usually assumed that the first publication of an "anomaly" in the academic literature is of great relevance (Schwert, 2003; Marquering et al., 2006; Jacobsen and Visaltanachoti, 2009). As a result, two issues are important when testing the potential success of a trading strategy: 1) the availability of adequate investment instruments, which makes it possible to effectively implement the strategy, and 2) the date that a trading rule became publicly known and accepted in the investment community. Given that the "Sell in May and Go Away" strategy is easy to implement and very popular among private and institutional investors, there is naturally a question of how it can continue to offer such potential for outperformance. Several recent studies have addressed this phenomenon (Swinkels and van

Vliet, 2012; Jacobsen and Zhang, 2012; Zhang and Jacobsen, 2012; Andrade et al., 2013). Most notably, Dichtl and Drobetz (2014) were not able to confirm that the Halloween strategy outperformed a buy-and-hold strategy or any other monthly seasonality-based strategy.

We implement a bootstrap-based simulation framework to test whether the Halloween strategy remains good advice for investors. In contrast to Dichtl and Drobetz (2014), we follow the methodological approach of prior studies, and compare it with only a buy-and-hold benchmark, rather than with all other monthly seasonality-based strategies. In addition to analyzing the time period during which adequate investment vehicles have been available to effectively implement the strategy, we also consider the period when the effect was first documented in a top academic journal (Bouman and Jacobsen, 2002).[1]

We believe our approach will enable us to exploit small datasets in the most efficient possible way. In existing studies, the investment horizon is determined by the length of the available dataset (which results in unrealistically long investment horizons in most cases). We set the investment horizon to one year, because even long-term investors tend to evaluate their portfolios on a yearly basis due myopic loss aversion (Benartzi and Thaler, 1995; Barberis et al., 2001). Furthermore, we implement hypothesis tests from which we can derive conclusions about statistical significance. Our hypothesis tests, which are also based on bootstrap simulations, do not require distributional assumptions for the return and risk measures.

As Jones and Lundstrum (2009) note, in order to ensure backtests are realistic, it is important to use only historical data that was available at the time an investment strategy was implemented. Backtesting further requires that instruments with sufficient market liquidity are available during the entire sample period (e.g., index funds or exchange-traded funds).

[1] Jacobsen and Visaltanachoti (2009) assume April 15, 1998 as the publication date, when the first draft of Bouman and Jacobsen's (2002) study was distributed through the SSRN network. However, we believe this study received public attention when it was published in the *American Economic Review* later in 2002. Most articles in practical investment journals and newspapers cite that version of the study.

Therefore, in contrast to Bouman and Jacobsen's (2002) study, as well as more recent studies such as Andrade et al. (2013) and Jacobsen and Zhang (2012), we do not use the standard MSCI stock market indices (which begin in 1970 for most developed stock markets). Instead, we work with return data from stock market indices that are easy to invest in and feature low transaction costs. For example, the German stock market index DAX consists of only thirty liquid blue-chip stocks, thus enabling full replication at low cost through efficient basket trades. A passive DAX index fund has been available since 1992, and a liquid DAX futures contract has been traded since November 1990. The MSCI Germany stock market index, on the other hand, is comprised of fifty-two constituents. To the best of our knowledge, no index fund or futures contract with a long enough history is available. Furthermore, in contrast to many other studies that focus solely on the U.S. market (Maberly and Pierce, 2004; Jones and Lundstrum, 2009; Witte, 2010; Swinkels and van Vliet, 2012), we follow Bouman and Jacobsen (2002) and implement our analyses in an international context.

Our study thus differs from prior research along several important dimensions. First, we focus on markets and time periods that allow an effective implementation of the Halloween strategy. Second, we consider the point at which the strategy was first published in a top academic journal (as a representation of public knowledge). Third, because our approach requires relatively short return series, we use a bootstrap simulation approach, which mitigates potential data-snooping problems (Lo and MacKinlay, 1990; Sullivan et al., 2001). Fourth, to incorporate the myopic loss aversion property of most investors, we evaluate strategy outcomes on a yearly basis. Fifth, our setup enables us to conduct statistical hypothesis tests. Sixth, and finally, we perform all analyses in an international context.

Our results are twofold. First, in line with prior studies, we confirm the existence of a Halloween anomaly when we use the maximum history of available index data for our analyses. This finding is based on a simple regression model, the method of choice in most related studies, and on our bootstrap simulation approach. However, when we consider the availability of adequate investment instruments, as well as the publication date of the seminal Bouman and Jacobsen (2002) study, we find evidence that the Halloween effect has become weaker and even diminished over time. While this finding is in sharp contrast to other recent studies, it is in line with the theory of market efficiency (Fama, 1970, 1991; Jensen, 1978).

The remainder of this paper is structured as follows. Section 2 gives a literature overview, while section 3 describes our data. Section 4 describes the results of our regression analyses. Section 5 introduces the design of our bootstrap-based simulation and discusses the results. Section 6 concludes and discusses implications for private and institutional investors.

2. Literature review

Bouman and Jacobsen (2002) was the first scientific paper published in a major academic journal that analyzed the Halloween effect. The authors document the “Sell in May” effect for thirty-six of thirty-seven sample countries. Their study uses linear regression models with dummy variables, but it also tests trading strategies. They find that the Halloween trading strategy provided lower returns than the buy-and-hold strategy for only two out of eighteen countries. However, in terms of volatility, the Halloween strategy dominated the buy-and-hold benchmark in all cases.

Maberly and Pierce (2004) argue that Bouman and Jacobsen’s (2002) regression results may be caused by data outliers. They compare the Halloween strategy based on S&P 500 future contracts with a buy-and-hold strategy, and reject the hypothesis that the Halloween effect offers a profitable trading rule. However, Witte (2010) criticises Maberly and Pierce’s (2004) regression setup. He shows that if data outliers are handled using a robust regression methodology, instead of simply eliminating single outliers, Maberly and Pierce’s (2004) conclusions cannot be confirmed.

Lucey and Zhao (2008) analyze the Halloween effect in the U.S. stock market using monthly CRSP Stock File Capitalization Decile Indices. They conclude that evidence for the Halloween effect is weak, and attribute it more to the January effect. Jones and Lundstrum (2009) claim that a test based on trading strategies is more appropriate than a mere statistical analysis (e.g., estimation of regression models). In particular, they note it is important to focus on stock markets and time periods with liquid ETFs and futures contracts. They also find that the Halloween trading strategy is not superior when based on the Vanguard S&P 500 index fund. Jacobsen and Visaltanachoti (2009) test the Halloween effect for U.S. sectors. They observe the effect for more than two-thirds of the sectors and industries studied, and find it can be exploited to improve an investor’s risk-return trade-off within a sector rotation strategy.

Dzhabarov and Ziemba (2010) also include the Halloween effect in their comprehensive study of seasonal anomalies in the U.S. stock market (based on Russell 2000 and S&P 500 futures). In contrast to their findings for most other anomalies, they conclude that the Halloween effect continues to exist. Moreover, Zhang and Jacobsen (2012) also find a Halloween effect in their long-run data, but they emphasize that it fluctuates over time (as do other seasonal effects), and varies strongly depending on sample size. Swinkels and van Vliet (2012) test the interactions of various well-known calendar effects. They conclude that the “turn of the month” effect and the Halloween effect are the strongest. In their backtests, a Halloween strategy dominates the passive stock investment in all cases in terms of return, volatility, and Sharpe ratio.

Andrade et al. (2013) implement an out-of-sample update of Bouman and Jacobsen’s (2002) study based on several international MSCI stock market indices. They confirm Bouman and Jacobsen’s (2002) earlier regression results. However, their trading strategy is based solely on the S&P 500 index, and the Halloween strategy generally provides higher returns than a passive S&P 500 investment. But in all cases (with and without leverage), the Halloween strategy dominates the buy-and-hold strategy in terms of the Sharpe ratio. Furthermore, all tested Halloween strategies provide a lower beta and a statistically significant alpha against a passive S&P 500 investment. Andrade et al. (2013) conclude that "the adage Sell in May and Go Away remains good investment advice."

Jacobsen and Zhang (2012) use all available stock market data for the 108 countries that have a stock market. They conclude that investors who exploit the Halloween effect achieved higher risk-adjusted returns than buy-and-hold investors even after the publication of Bouman and Jacobsen's (2002) study. They classify the Halloween effect as "a strong market anomaly that has strengthened rather than weakened in the recent years." Jacobsen and Zhang (2012) also use price index data and argue that dividend payments do not affect their results if there is no clustering in a specific month.[2] While this notion applies in a regression framework, however, it no longer holds when investment strategies are implemented. A buy-and-hold investor receives dividends for all twelve months, while an investor applying the Halloween strategy would only receive dividends for six months from November through April. This effect is correctly captured when simulations are based on performance indices, but it is ignored in simulations with price indices. In fact, the use of price indices implies that the buy-and-hold investment strategy is at a disadvantage against the Halloween strategy. Zhang and Jacobsen (2012) omit transaction costs in their simulations, which also adversely affects the buy-and-hold benchmark performance compared to the Halloween strategy.

In summary, all of the above-mentioned studies suffer from some or all of the following shortcomings: 1) They focus only on the U.S. stock market, 2) they do not simulate a realistic enough Halloween investment strategy, 3) they provide no statistical inference, 4) they use all

[2] See Jacobsen and Zhang (2012, p. 9, footnote 3). The same argument is presented in Gultekin and Gultekin (1983), Bouman and Jacobsen (2002), and Zhang and Jacobsen (2012).

available index data regardless of whether adequate investment instruments are available, and 5) they fail to consider when the Halloween effect was first published in a major academic journal. We believe our simulation setup addresses all these issues appropriately.

3. Data description

To ensure our analysis is relevant for practical investing, we use monthly return data of liquid stock markets that are investable with low transaction costs. As we note above, in comparison to a buy-and-hold strategy, no dividend payments are received under the Halloween strategy from May through October. By using total return indices (with dividend payments reinvested), we adequately incorporate this effect into our analysis.

In contrast to Bouman and Jacobsen (2002), Andrade et al. (2013), and Jacobsen and Zhang (2012), we do not use MSCI stock market indices here (with one exception). Jones and Lundstrum (2009) emphasize that investors are unlikely to use MSCI indexes in their asset allocation strategy even today, and they were unable to do so in 1970 (which is the starting date of Bouman and Jacobsen's (2002) sample). They further argue that trading strategy results based on MSCI indexes are of limited practical relevance. Therefore, we use popular stock market indices for which liquid investment instruments were available: the S&P 500 TR index for the U.S., the DAX 30 TR index for Germany, the FTSE 100 TR index for the U.K., and the CAC 40 TR index for France.[3] We also include the EuroStoxx 50 NR index and the MSCI Emerging Markets TR index. The latter MSCI index is the most established and frequently used index for broad stock investments in emerging markets. Total return indices are used in all cases (except the EuroStoxx 50 Net Return; see Exhibit 1).

[3] In contrast to Jones and Lundstrum (2009), we work with index data, not real funds data. Jacobsen and Visaltanachoti (2009) note that using real funds data has several shortcomings compared to index data (see their footnote 18).

In contrast to most prior studies, we explicitly consider when an investment in a given stock market index became possible for both institutional and private investors. Because futures investments are generally not allowed for private or institutional investors (e.g., due to regulatory restrictions), we focus primarily on the availability of passive mutual index funds or Exchange Traded Funds (ETFs).[4] For example, in 1992, the bank Sal. Oppenheim jr. & Cie. KGaA launched a passively managed index fund for the DAX, the German stock market index (Ebertz and Ristau, 1992). Liquid futures contracts have been available on the DAX since November 1990. Therefore, we can assume that DAX investments have been available for almost all institutional and private investors since 1993. In the U.S., the Vanguard S&P 500 Index fund has been available since 1976 (i.e., much longer than the total return version of the S&P 500 index, which began trading on December 31, 1987).[5] For all other countries or regions, we similarly identify the date on which investment instruments that would effectively allow the implementation of a Halloween strategy became available, not only to institutional investors but also for most private investors (see Exhibit 1).

[Insert Exhibit 1 here]

Following existing studies, we use a one-month interest rate as the risk-free asset (Jacobsen and Visaltanachoti, 2009; Swinkels and van Vliet, 2012). The corresponding cash market indices are listed in Exhibit 1. To ensure that we have exactly the same number of months ranging from May through October and from November through April, we begin all index data in January (and end in December). Therefore, we work with full years of data, with exactly six months invested in the stock market, and six months in cash.

[4] In most cases, futures contracts were available before the first index funds were launched. For example, futures on the DAX have been available since November 1990. However, buying and selling futures contracts was not or not easily possible for most private investors in Germany. Additionally, regulatory restrictions or statutes allowed many institutional investors to buy or sell futures only for hedging purposes, not for active trading.

[5] Because our analysis is based on index data instead of real funds data, for the U.S. we use 1988 as our beginning year.

4. Linear regression analysis

4.1. Main regression results

In this section, we implement Bouman and Jacobsen's (2002) regression analysis (which has been replicated in many follow-up studies). All regressions are estimated using continuously compounded monthly returns of the stock market indices shown in Exhibit 1. In particular, we estimate the following model using a standard OLS technique:

(1) $$R_t = \mu + \alpha_1 \times S_t + \varepsilon_t,$$

where R_t denotes the stock market return at time t, and S_t is a dummy variable that is equal to 1 if month t falls within the time interval from November to April, and 0 otherwise. μ is the regression intercept, and ε_t is an error term. To consider a possible January effect, we also estimate the following extended regression model:

(2) $$R_t = \mu + \alpha_1 \times S_t^{adj} + \alpha_2 \times J_t + \varepsilon_t,$$

where the dummy variable S_t^{adj} is equal to 1 if month t falls within the time interval from November to April (January is the only exception). In January, S_t^{adj} is equal to 0. S_t^{adj} is also equal to 0 in all months from May to October. The dummy variable J_t controls for a possible January effect; this variable is equal to 1 in January, and 0 otherwise.

In a first step, we estimate both regression models in (1) and (2) using all available index data. All index return data end in December 2012, and have an index-specific start date (see Exhibit 1). The results are summarized in panel A of Exhibit 2. In a second step, we divide the full sample into two subsamples. Subsample 1 captures the availability of suitable investment instruments. The regression results are shown in panel B of Exhibit 2. Subsample 2 captures the publication date of Bouman and Jacobsen's (2002) study, thus panel C presents the regression results when all index data begin as late as January 2003. The sample period ends in December 2012 in all models.[6]

[Insert Exhibit 2 here]

Using all available index data in regression model 1, we find a positive (and statistically significant) α_1 estimate for all countries or regions in panel A of Exhibit 2. Accordingly, the monthly returns from November through April are significantly higher than the corresponding returns from May through October, which is usually interpreted as evidence for the Halloween effect. Controlling for a potential January effect in model 2, our results are unchanged. The estimated α_1 remains positive and statistically significant in all countries or regions. Overall, these results confirm the existence of the Halloween effect when all available index data is used.

As panel B shows, however, our results become much weaker when we consider the availability of adequate investment instruments. Based on a 5% significance level, we observe a significant Halloween effect in only two of six regressions based on model 1 (the S&P 500 and the DAX 30).[7] In model 2, the estimated α_1 also becomes significant for the FTSE 100. As indicated by the negative α_2 coefficient (which is nearly significant), this latter observation is attributable to a reverse January effect. We find no significant Halloween effect for the EuroStoxx 50, the CAC 40, or the MSCI Emerging Markets in either regression model specification.

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[6] In general, we correct the t-statistics for heteroscedasticity using White's (1980) method. In the presence of autocorrelated residuals, however, we use Newey and West's (1987) method instead.

[7] In contrast to all other stock markets, the U.S. market did have an adequate investment instrument available before the total return version of the S&P 500 was introduced. Note that, because we use index data here rather than real funds data (Jacobsen and Visaltanachoti, 2009), both models in panels A and B deliver identical results for the S&P 500. A similar argument applies for the results of the MSCI Emerging Markets model in panels B and C.

Finally, when we account for the publication date of Bouman and Jacobsen (2002) and begin our sample period as late as January 2003 (going through December 2012), the results in panel C show that all significant α_1 coefficients vanish in both model specifications (except for the FTSE 100 in model 2). Accordingly, the Halloween effect is no longer observable during this latest subperiod.

4.2 Robustness tests

To verify the robustness of our results, we follow Jacobsen and Zhang (2012) and repeat our regressions by modeling the residuals as a GARCH (1,1) process. In particular, we specify the residuals in Equations (1) and (2) as follows:

(3) $\varepsilon_t|\Phi_{t-1} \sim N(0; \sigma_t^2)$ and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \sigma_{t-1}^2.$

Exhibit 3 shows our results for the regressions with GARCH (1,1) residuals. Except for the S&P 500 index, we observe a significant Halloween effect in all stock markets when the full history of index data is used (panel A). When we incorporate the availability of index funds, we continue to observe the Halloween effect on the German market during this shorter sample period (panel B). For the FTSE 100 index, we observe a significant Halloween effect only in model 2, which controls for a potential January effect.

Most importantly, the results in panel C of Exhibit 3 are qualitatively the same as in panel C of Exhibit 2. As with the OLS estimation, the regression model using GARCH (1,1) residuals indicates that the Halloween effect vanished after Bouman and Jacobsen (2002) was published (i.e., during the most recent 2003-2012 sample period).

[Insert Exhibit 3 here]

To analyze the influence of outliers, we repeat our analysis by running median regressions. The mean value in a traditional OLS estimation minimizes the sum of squared residuals; the median value minimizes the sum of absolute residuals (Koenker and Hallock, 2001a, 2001b). As a result, median regressions are more robust against the influence of outliers. Exhibit 4 summarizes our median regression results. Even when we use all available index data (panel A), we see that the Halloween effect is now much less pronounced than for the OLS estimation. This result holds for the S&P 500, the DAX 30, and the MSCI EM, where the estimated α_1 coefficient is no longer statistically significant. As expected, we find no significant coefficients for the shorter subsamples in panels B and C (with two exceptions when model 2 is applied). These results confirm Maberly and Pierce's (2004) conjecture that the Halloween effect is (at least partly) driven by some extreme monthly return observations.

[Insert Exhibit 4 here]

In summary, our regression analyses corroborate the existence of the Halloween effect when all available index data are used. However, even for the longest sample periods, we find that, in some cases, the Halloween effect is driven by a few extreme monthly return observations. In addition, when we account for the availability of adequate investment instruments, and the publication date of Bouman and Jacobsen's (2002) study, the Halloween effect weakened strongly or even diminished.

5. Bootstrap-based simulation of trading strategies

A stronger test for seasonal anomalies could analyze their potential to earn abnormal returns. Therefore, following Jones and Lundstrum (2009), we test the profitability of trading strategies based on the Halloween effect in order to derive further conclusions about market efficiency. We first describe our simulation framework (section 5.1), and then discuss the simulation setup used for statistical inference (section 5.2). Finally, we present and discuss our trading strategy simulation results (section 5.3) and perform robustness tests (section 5.4).

5.1. Simulation analysis design

All the prior studies discussed in section 2 used trading strategies based on the Halloween effect as traditional historical backtests, implying that only a single return path was considered. To overcome this deficiency, and to use the data in the most efficient way, we implement a bootstrap simulation approach instead. In particular, we divide all available monthly returns for a country or region into two subsets: Subset 1 contains the monthly stock and cash market returns from May through October, and subset 2 contains the returns from November through April. Following Barberis et al. (2001), we set the investment horizon to one year, recognizing that even long-term investors tend to evaluate their portfolio performance on a yearly basis (Benartzi and Thaler, 1995).

To simulate the Halloween and buy-and-hold strategies, we draw (with replacement) six pairs of monthly stock and cash market returns from subset 1, and six pairs of monthly returns from subset 2. We use the first four monthly stock market returns from subset 2 as the returns for the Halloween strategy and the buy-and-hold from January through April. The six monthly cash market returns from subset 1 are used for the Halloween strategy from May through October, and the corresponding stock market returns are used for the buy-and-hold. Finally, we use the remaining two monthly stock market returns from subset 2 for both strategies for November and December. The shift from the stock market into the cash market in May, and the reverse shift in October under the Halloween strategy are executed with transaction costs of 50 bps (DeMiguel et al., 2009).

This procedure is repeated 100,000 times, resulting in 100,000 yearly returns for the Halloween strategy and the buy-and-hold benchmark.[8] We transform all returns into excess returns by subtracting the risk-free rate.[9] For both strategies, we calculate the mean annual

[8] Repeated simulations indicate that our results are sufficiently stable with this number of runs.
[9] Because we draw pairs of monthly stock and cash market returns, this transformation is straightforward.

excess return, the volatility of annual excess returns, and the Sharpe ratio.[10] As in our regression analysis in section 4, we again simulate our investment strategies using the full index history as well as the two shorter subsamples (accounting for the availability of adequate investment instruments and the publication date of Bouman and Jacobson's (2002) study, respectively).

To further verify the robustness of our bootstrap simulation results, we modify the simulation design. Instead of dividing the available data into two subsamples, we generate twelve subsamples, one for each month. For example, subset 1 contains all return pairs for January, subset 2 contains all return pairs for February, and so on. For each month, we draw a pair of stock and cash market returns from the underlying subsample with replacement. For the Halloween strategy, we use the stock market returns from January through April and November and December, and the cash market returns from May through October. For the buy-and-hold strategy, we use the monthly stock market return for each of the twelve months. In comparison with our original bootstrap design, this alternative guarantees that the monthly structure of any given year is maintained, i.e., each year contains a January return, followed by a February return, and so on.

As an additional robustness check, we reduce the turnover-based round-trip transaction costs to 0.1% (Solnik, 1993) and then increase them to 1.0% (Pesaran and Timmermann, 1994). As proposed by Andrade et al. (2013), we also implement a "leveraged Halloween strategy," which, just as in the traditional Halloween strategy, also holds cash from May through October. However, a 200% stock market allocation is accumulated from November through April, implying a 100% stock allocation over the entire year on average (similarly to the buy-and-hold benchmark strategy).

[10] Following Sharpe (1994), we compute the Sharpe ratio as the ratio of the mean excess return to the volatility of the excess return.

5.2. Statistical significance testing

Our simulation framework allows us to implement statistical significance tests for the performance differences between the Halloween and buy-and-hold strategies. Our starting points are the $N = 100{,}000$ yearly returns for the two strategies. To evaluate the profitability of the Halloween strategy, we focus on the differences among the mean excess returns, the volatilities of the excess returns, and the Sharpe ratios of the Halloween strategy H and the buy-and-hold benchmark strategy B:[11]

(4) $\Delta_{\Pi} = \Pi_H - \Pi_B.$

We aim to analyze whether there is a statistically significant difference in the performance measure Π (mean excess return, volatility, or Sharpe ratio) of the two compared strategies (Π_H, Π_B). Formally, we test:

(5) $H_0: \Delta_{\Pi} = 0$ against $H_1: \Delta_{\Pi} \neq 0.$

Given the $N = 100{,}000$ yearly returns for each strategy, an appropriate point estimator for the performance difference in (4) is:

(6) $\widehat{\Delta}_{\Pi} = \widehat{\Pi}_H - \widehat{\Pi}_B.$

In order to implement a statistical significance test, we require the distribution of the $\widehat{\Delta}_{\Pi}$ value. We thus run another bootstrap simulation based on the $N = 100{,}000$ yearly returns (i.e., the result of the preceding strategy simulation bootstrap). Within this second bootstrap, we create 1,000 bootstrap resamples ($N_B = 1{,}000$), each consisting of 1,000 yearly returns.[12] We compute the $\widehat{\Delta}_{\Pi}^{*}$ value for each $N_B = 1{,}000$ bootstrap resamples in the same way as for the $\widehat{\Delta}_{\Pi}$ value. Let

[11] Zeisberger et al. (2007) implement a similar type of hypothesis testing based on a bootstrap approach.
[12] The results of our hypothesis tests are stable with this number of simulation runs.

(7) $\widehat{\Delta}^*_{\Pi[1]} \leq \widehat{\Delta}^*_{\Pi[2]} \leq \cdots \leq \widehat{\Delta}^*_{\Pi[N_B]}$

equal the ordered series of the differences in the performance measures. Based on this series, we can construct the following confidence interval as per Efron and Tibshirani (1998):

(8) $CI = \left[\xi^*_{low}, \xi^*_{high}\right]$

where:

(9a) $\xi^*_{low} = \widehat{\Delta}^*_{\Pi[\alpha/2 \cdot N_B]}$ and (9b) $\xi^*_{high} = \widehat{\Delta}^*_{\Pi[(1-\alpha/2) \cdot N_B]}$.

We can reject the null hypothesis H_0 at significance level α if $0 \notin CI$. We use Efron's (1979) standard bootstrap technique because our bootstrap sample consisting of 100,000 elements does not exhibit any significant serial dependencies. As described in section 5.1, we obtain the 100,000 yearly returns of the Halloween strategy and the buy-and-hold benchmark by randomly drawing monthly returns.[13]

Exhibit 5 illustrates our simulation setup, which consists of the first-step bootstrap for the simulation of investment strategies, and the second-step bootstrap for the hypothesis tests. Let R_i be a pair of yearly returns of the Halloween strategy H and the buy-and-hold benchmark B: $R_i = (R_{Hi}; R_{Bi})$. The second-stage bootstrap uses the N pairs of annual returns $(R_1, \dots, R_N)$ from the first-stage bootstrap, with point estimator $\widehat{\Delta}_\Pi = \Delta_\Pi(R_1, \dots, R_N)$. Similarly, the $\widehat{\Delta}^*_\Pi$ values are $\widehat{\Delta}^*_\Pi = \Delta_\Pi\left(R^*_1, \dots, R^*_{1,000}\right)$, where the R^*_i values refer to the results from the second-stage bootstrap (statistical inference bootstrap).

[Insert Exhibit 5 here]

[13] Note that overlapping blocks of data or even blocks of data from a rolling window approach would suffer from serial dependencies. Politis and Romano (1994) suggest a stationary block bootstrap approach that is appropriate even with weakly dependent data. As a robustness check, we implement their approach and use Patton et al.'s (2009) methodology to determine the optimal (average) block length. We find that a block length of 1 – and thus the application of Efron's (1979) standard bootstrap approach – is appropriate in our context.

5.3. Main simulation results

Exhibit 6 presents the results of our baseline bootstrap simulations, where we draw twelve monthly returns from the two subsets and assume transaction costs of 50 bps (see section 5.1.). Panel A of Exhibit 6 shows the results with all available index data.[14] In five of the six markets, the Halloween strategy provided higher excess returns than the buy-and-hold. In one case (the German DAX), the difference is even statistically significant at the 1% level. In contrast to buy-and-hold, the Halloween strategy is not fully invested in the stock market during the assumed investment horizon of one year. It thus exhibits significantly lower excess return volatilities in all six countries and regions. Finally, based on the Sharpe ratio, the Halloween strategy generates significantly higher risk-adjusted excess returns than the buy-and-hold in all cases.

When we consider the availability of adequate investment instruments in panel B of Exhibit 6, we find similar results. The Halloween strategy provides lower excess returns than the buy-and-hold for the S&P 500 and the MSCI Emerging Markets, but its excess returns are higher for the EuroStoxx 50, the DAX 30, the CAC 40, and the FTSE 100. Moreover, because of its lower volatility, the Halloween strategy dominates buy-and-hold in terms of the Sharpe ratio in all six countries (the differences are statistically significant).

[Insert Exhibit 6 here]

Finally, panel C of Exhibit 6 shows the results for simulating both strategies after the publication of Bouman and Jacobsen’s (2002) study (i.e., the most recent 2003-2012 sample period). We now observe that the Halloween strategy exhibits statistically significant lower

[14] In contrast to our regression analysis in section 4, here the maximum length of available index data is determined not only by stock market data but also by cash market data. For example, monthly stock market data are available from 1965 for the German DAX. The regressions including all index data begin in January 1965 and end in December 2012. However, cash market data are only available from 1991, so our trading strategy simulations using all available index data only begin in January 1991 (and end in December 2012).

excess returns than buy-and-hold for all six countries or regions. Comparing Sharpe ratios, the Halloween strategy significantly dominates buy-and-hold only for the MSCI Emerging Markets index. In all other countries or regions, either the buy-and-hold strategy dominates or the Sharpe ratio differences are no longer statistically significant. Overall, our simulation results do not support the hypothesis that the Halloween effect continues to offer a safe "free lunch" for investors.

5.4. Robustness tests

To check the robustness of our results, we repeat the bootstrap simulations using the modified simulation design as described in section 5.1. Instead of drawing returns from two subsets of data, we draw the return pairs from the twelve different monthly subsets. The results shown in Exhibit 7 generally confirm our baseline simulation results. The Halloween strategy dominates buy-and-hold in terms of Sharpe ratios in panels A and B. However, the results in panel C are again markedly different. In terms of Sharpe ratios, the Halloween strategy significantly dominates the buy-and-hold benchmark only for the MSCI Emerging Markets index. In all other countries or regions, buy-and-hold either generates significantly higher Sharpe ratios, or the difference is lost in estimation errors.

[Insert Exhibit 7 here]

In another robustness check, we examine the influence of different levels of transaction costs. Exhibit 8 shows our results with varying transaction costs for the last ten sample years (January 2003 through December 2012).[15] The results for low transaction costs of 0.1% (round-trip) are summarized in panel A of Exhibit 8. As expected, lower transaction costs make the Halloween strategy more attractive than the buy-and-hold strategy. However, even with low transaction costs, the Halloween strategy provides significantly lower excess returns

[15] The results in Exhibit 8 are based on bootstrap simulations where we draw the monthly returns from the twelve data subsets. The results should therefore be compared with those in Exhibit 7.

than buy-and-hold in all six countries or regions. Note further that the performance of the Halloween and buy-and-hold strategies for the EuroStoxx 50 is now similar in terms of Sharpe ratios. For the FTSE 100, the Halloween strategy even exhibits a significantly higher Sharpe ratio. For the DAX 30, the Halloween strategy now becomes superior to buy-and-hold, albeit the difference is not statistically significant.

The results for high transaction costs of 1.0% are shown in panel B of Exhibit 8. As expected, the dominance of buy-and-hold in terms of excess returns increases dramatically. Except for the MSCI Emerging Markets index, buy-and-hold now significantly outperforms the Halloween strategy in terms of Sharpe ratios.

[Insert Exhibit 8 here]

In a final robustness check, we implement a leveraged version of the Halloween strategy (Andrade et al., 2013), whereby no stocks are held from May through October and a leveraged position of 200% stocks is accumulated from November through April. The simulations are implemented again over the most recent January 2003-December 2012 sample period. Panel A in Exhibit 9 gives the results, assuming transaction costs of 0.50% (the base case). As expected, leveraging the Halloween strategy results in higher excess returns but also higher volatilities compared to buy-and-hold. Except for the EuroStoxx 50 and the CAC 40, we observe significantly higher excess returns for the Halloween strategy than for the buy-and-hold. However, note that, in all cases, the leveraged Halloween strategy exhibits significantly higher volatilities. This volatility effect implies that the higher excess returns of the Halloween strategy are overcompensated in simulations with the DAX 30 and the FTSE 100, because in both cases the leveraged Halloween strategy is dominated by the buy-and-hold strategy in terms of Sharpe ratios (for the FTSE 100, this difference is not statistically significant). When we reduce the transaction costs to the lower 0.1% rate, the leveraged Halloween strategy becomes more attractive than the passive benchmark in panel B in Exhibit 9. The Halloween strategy exhibits a significantly higher Sharpe ratio than the buy-and-hold strategy for the S&P 500 and the FTSE 100. However, buy-and-hold still dominates in terms of Sharpe ratios for the EuroStoxx 50 and the CAC 40 even with low transaction costs (although the differences are again not significant).

Overall, we find that leveraging may allow us to enhance the return potential of the Halloween strategy, but at the cost of substantially higher volatility. However, it is important to consider that not all institutional or private investors are allowed to implement leveraged investment strategies (e.g., due to regulatory restrictions and solvency restraints).

[Insert Exhibit 9 here]

6. Concluding remarks

This study examines the long-standing "Sell in May and Go Away" investment strategy based on the Halloween effect. We use a full sample with the maximum length of historical index data, and find that both regression analyses and tests of bootstrap-simulated investment strategies confirm the existence of the Halloween effect. These findings are in line with prior studies.

However, when we explicitly consider the time period during which adequate investment instruments were available to implement the Halloween strategy, and further take into account the publication date of Bouman and Jacobsen's (2002) seminal study, our results provide strong evidence that the Halloween effect has weakened or even diminished recently. In fact, we do not believe this simple and well-known investment rule, which nowadays can be implemented by most private and institutional investors, constitutes an ongoing "free lunch."

Our results are in line with the theory of efficient capital markets. Therefore, investors should only consider applying the Halloween strategy in the future with caution.

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Exhibit 1: Availability of index data and investment instruments

Country/region	Index	Availability	Availability of stock market fund
United States	S&P 500 TR 1-month T-Bills (CRSP)	1988 1927	1976 (Vanguard 500 Index)
Europe	EuroStoxx 50 NR Euribor/Fibor 1M	1987 1991	2001 (iShares EuroStoxx 50)
Germany	DAX 30 Performance Euribor/Fibor 1M	1965 1991	1993 (Oppenheim DAX-Werte-Fonds)
France	CAC 40 TR PIBOR 1M	1988 1988	2001 (Lyxor ETF CAC 40)
United Kingdom	FTSE 100 TR Libor 1M	1986 1986	2001 (iShares FTSE 100)
Emerging Markets	MSCI EM TR 1-month T-Bills (CRSP)	1988 1927	2004 (iShares MSCI EM ETF)

The table summarizes the availability of index data and investment instruments. Index funds were launched in the year prior to the date listed in the table. All indices are on a total return basis. An exception is the EuroStoxx 50 Net Return, which became available in January 1987, while the total return version of this index was only introduced in February 2001. The Euribor 1M rate is available since 1999, thus we use the Fibor 1M rate for the earlier sample years.

Exhibit 2: Standard dummy variables regression approach

	Regression model 1 (without January effect)		Regression model 2 (with January effect)		
Country/region	μ t-value / prob.	α_1 t-value / prob.	μ t-value / prob.	α_1 t-value / prob.	α_2 t-value / prob.
Panel A: Period I (availability of stock market index data – 12/2012)					
S&P 500	0.0034 0.90 / 0.37	0.0086 1.93 / 0.05	0.0034 0.90 / 0.37	0.0099 2.13 / 0.03	0.0021 0.24 / 0.81
EuroStoxx 50	-0.0019 -0.37 / 0.71	0.0150 2.53 / 0.01	-0.0019 -0.37 / 0.71	0.0175 2.94 / 0.00	0.0027 0.24 / 0.80
DAX 30	-0.0020 -0.52 / 0.60	0.0135 2.98 / 0.00	-0.0020 -0.52 / 0.60	0.0130 2.77 / 0.01	0.0161 1.78 / 0.08
CAC 40	-0.0004 -0.07 / 0.94	0.0145 2.38 / 0.02	-0.0004 -0.07 / 0.94	0.0171 2.74 / 0.01	0.0013 0.10 / 0.92
FTSE 100	0.0014 0.34 / 0.73	0.0124 2.64 / 0.01	0.0014 0.34 / 0.73	0.0150 3.22 / 0.00	-0.0001 -0.01 / 0.99
MSCI EM	-0.0013 -0.18 / 0.86	0.0023 2.62 / 0.01	-0.0013 -0.18 / 0.86	0.0239 2.72 / 0.01	0.0161 1.01 / 0.31
Panel B: Period II (availability of stock market funds – 12/2012)					
S&P 500	0.0034 0.90 / 0.37	0.0086 1.93 / 0.05	0.0034 0.90 / 0.37	0.0099 2.13 / 0.03	0.0021 0.24 / 0.81
EuroStoxx 50	-0.0067 -0.78 / 0.43	0.0097 1.01 / 0.32	-0.0067 -0.78 / 0.44	0.0136 1.45 / 0.15	-0.0101 -0.53 / 0.59
DAX 30	-0.0024 -0.38 / 0.71	0.0182 2.18 / 0.03	-0.0024 -0.36 / 0.72	0.0212 2.57 / 0.01	0.0031 0.21 / 0.84
CAC 40	-0.0055 -0.67 / 0.50	0.0100 1.07 / 0.29	-0.0055 -0.67 / 0.50	0.0131 1.44 / 0.15	-0.0058 -0.32 / 0.75
FTSE 100	-0.0022 -0.33 / 0.74	0.0094 1.28 / 0.20	-0.0022 -0.33 / 0.74	0.0154 2.14 / 0.03	-0.0203 -1.50 / 0.14
MSCI EM	0.0020 0.13 / 0.89	0.0167 1.06 / 0.29	0.0020 0.13 / 0.89	0.0213 1.38 / 0.17	-0.0067 -0.22 / 0.82
Panel C: Period III (01/2003 – 12/2012)					
S&P 500	0.0022 0.31 / 0.75	0.0071 0.95 / 0.34	0.0022 0.31 / 0.75	0.0112 1.53 / 0.13	-0.0133 -0.96 / 0.34
EuroStoxx 50	0.0018 0.23 / 0.82	0.0030 0.33 / 0.74	0.0018 0.23 / 0.82	0.0073 0.85 / 0.39	-0.0186 -0.86 / 0.39
DAX 30	0.0039 0.48 / 0.63	0.0083 0.88 / 0.38	0.0039 0.48 / 0.63	0.0141 1.60 / 0.11	-0.0203 -0.84 / 0.40
CAC 40	0.0028 0.36 / 0.72	0.0033 0.37 / 0.71	0.0028 0.36 / 0.72	0.0067 0.81 / 0.42	-0.0141 -0.69 / 0.49
FTSE 100	0.0039 0.60 / 0.55	0.0051 0.69 / 0.49	0.0039 0.60 / 0.55	0.0123 1.73 / 0.09	-0.0311 -2.07 / 0.04
MSCI EM	0.0020 0.13 / 0.89	0.0167 1.06 / 0.29	0.0020 0.13 / 0.89	0.0213 1.38 / 0.17	-0.0067 -0.22 / 0.82

The table reports the results from the standard dummy variables regressions approach. The two models are described in Eqs. (1) and (2) in Section 4. μ denotes the estimated intercept term, α_1 the coefficient that indicates the Halloween effect, and α_2 the coefficient on the dummy variable that controls for the January effect. All available index data are used in Panel A. Panel B accounts for the availability of suitable investment instruments, and Panel C captures the publication date of Bouman and Jacobsen‘s (2002) study. The first line reports the estimated coefficient. The second line contains the t-value (left) and the corresponding p-value (right). In general, t-statistics are corrected for heteroscedasticity using White's (1980) method. In the presence of autocorrelated residuals, however, we use Newey and West's (1987) method instead.

Exhibit 3: Dummy variables regression approach with GARCH (1,1) residuals

	Regression model 1 (without January effect)		Regression model 2 (with January effect)		
Country/region	μ t-value / prob.	α_1 t-value / prob.	μ t-value / prob.	α_1 t-value / prob.	α_2 t-value / prob.
Panel A: Period I (availability of stock market index data – 12/2012)					
S&P 500	0.0067 2.10 / 0.04	0.0049 1.16 / 0.25	0.0067 2.10 / 0.04	0.0049 1.07 / 0.29	0.0049 0.71 / 0.48
EuroStoxx 50	0.0006 0.14 / 0.89	0.0150 2.24 / 0.03	0.0006 0.14 / 0.89	0.0164 2.24 / 0.03	0.0073 0.74 / 0.46
DAX 30	-0.0004 -0.14 / 0.89	0.0120 2.65 / 0.01	-0.0005 -0.17 / 0.87	0.0103 2.08 / 0.04	0.0200 2.68 / 0.01
CAC 40	0.0030 0.70 / 0.48	0.0127 1.95 / 0.05	0.0030 0.68 / 0.49	0.0139 1.98 / 0.05	0.0072 0.74 / 0.46
FTSE 100	0.0035 1.20 / 0.23	0.0098 2.10 / 0.04	0.0035 1.22 / 0.22	0.0124 2.38 / 0.02	-0.0039 -0.50 / 0.62
MSCI EM	0.0029 0.52 / 0.60	0.0200 2.46 / 0.01	0.0029 0.52 / 0.61	0.0194 2.38 / 0.02	0.0201 1.38 / 0.17
Panel B: Period II (availability of stock market funds – 12/2012)					
S&P 500	0.0067 2.10 / 0.04	0.0049 1.16 / 0.25	0.0067 2.10 / 0.04	0.0049 1.07 / 0.29	0.0049 0.71 / 0.48
EuroStoxx 50	0.0030 0.38 / 0.70	0.0060 0.56 / 0.57	0.0033 0.44 / 0.66	0.0094 0.89 / 0.37	-0.0119 -0.70 / 0.49
DAX 30	0.0029 0.61 / 0.54	0.0150 2.28 / 0.02	0.0029 0.47 / 0.64	0.0173 1.83 / 0.07	0.0038 0.30 / 0.76
CAC 40	0.0043 0.62 / 0.54	0.0088 0.95 / 0.34	0.0041 0.58 / 0.56	0.0107 1.16 / 0.25	-0.0036 -0.25 / 0.80
FTSE 100	0.0036 0.79 / 0.43	0.0077 1.07 / 0.28	0.0042 0.97 / 0.33	0.0112 1.64 / 0.10	-0.0148 -1.35 / 0.18
MSCI EM	0.0132 1.42 / 0.16	0.0078 0.59 / 0.55	0.0140 1.55 / 0.12	0.0119 0.86 / 0.39	-0.0165 -0.82 / 0.41
Panel C: Period III (01/2003 – 12/2012)					
S&P 500	0.0053 1.13 / 0.26	0.0061 1.00 / 0.32	0.0054 1.15 / 0.25	0.0076 1.19 / 0.23	-0.0022 -0.20 / 0.84
EuroStoxx 50	0.0065 0.85 / 0.39	0.0032 0.31 / 0.76	0.0067 0.87 / 0.38	0.0072 0.67 / 0.50	-0.0145 -0.88 / 0.38
DAX 30	0.0060 0.70 / 0.49	0.0099 0.90 / 0.37	0.0063 0.72 / 0.47	0.0150 1.14 / 0.25	-0.0147 -1.03 / 0.30
CAC 40	0.0069 1.00 / 0.32	0.0069 0.74 / 0.46	0.0067 0.96 / 0.34	0.0089 0.95 / 0.34	-0.0047 -0.34 / 0.74
FTSE 100	0.0062 1.39 / 0.17	0.0060 0.87 / 0.38	0.0066 1.57 / 0.12	0.0095 1.44 / 0.15	-0.0172 -1.68 / 0.09
MSCI EM	0.0132 1.42 / 0.16	0.0078 0.59 / 0.55	0.0140 1.55 / 0.12	0.0119 0.86 / 0.39	-0.0165 -0.82 / 0.41

The table reports the results from the standard dummy variables regressions approach using with GARCH (1,1) residuals. The two models are described in Eqs. (1) and (2) in Section 4, the specification of the residuals is shown in Equ. (3). μ denotes the estimated intercept term, α_1 the coefficient that indicates the Halloween effect, and α_2 the coefficient on the dummy variable that controls for the January effect. All available index data are used in Panel A. Panel B accounts for the availability of suitable investment instruments, and Panel C captures the publication date of Bouman and Jacobsen's (2002) study. The first line reports the estimated coefficient. The second line contains the t-value (left) and the corresponding p-value (right).

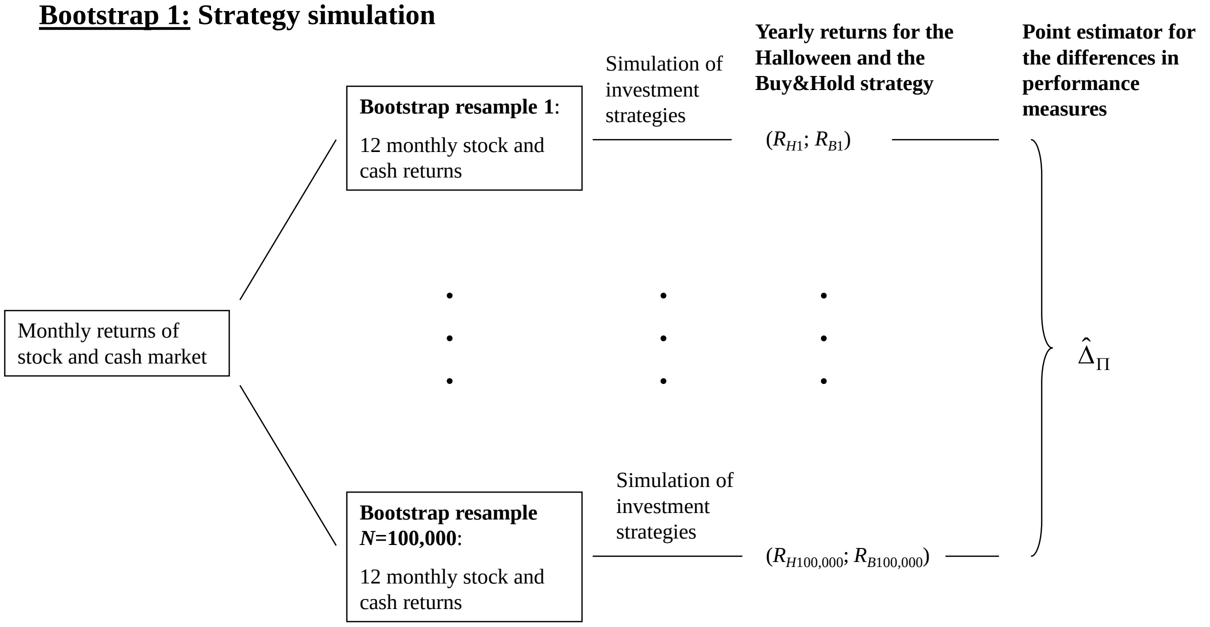
Exhibit 4: Median regressions

	Regression model 1 (without January effect)		Regression model 2 (with January effect)		
Country/region	μ t-value / prob.	α_1 t-value / prob.	μ t-value / prob.	α_1 t-value / prob.	α_2 t-value / prob.
Panel A: Period I (availability of stock market index data – 12/2012)					
S&P 500	0.0067 1.59 / 0.11	0.0071 1.25 / 0.21	0.0067 1.59 / 0.11	0.0069 1.18 / 0.24	0.0115 1.09 / 0.28
EuroStoxx 50	0.0073 1.38 / 0.17	0.0094 1.33 / 0.19	0.0073 1.37 / 0.17	0.0128 1.75 / 0.08	0.0051 0.36 / 0.72
DAX 30	0.0029 0.68 / 0.50	0.0085 1.58 / 0.11	0.0029 0.68 / 0.50	0.0065 1.17 / 0.24	0.0143 1.61 / 0.11
CAC 40	0.0091 1.58 / 0.12	0.0134 1.71 / 0.09	0.0091 1.57 / 0.12	0.0134 1.67 / 0.10	0.0134 0.78 / 0.43
FTSE 100	0.0079 1.87 / 0.06	0.0096 1.78 / 0.08	0.0079 1.87 / 0.06	0.0109 1.96 / 0.05	0.0005 0.05 / 0.96
MSCI EM	0.0091 1.52 / 0.13	0.0090 1.04 / 0.30	0.0091 1.52 / 0.13	0.0148 1.63 / 0.10	-0.0031 -0.19 / 0.85
Panel B: Period II (availability of stock market funds – 12/2012)					
S&P 500	0.0067 1.59 / 0.11	0.0071 1.25 / 0.21	0.0067 1.59 / 0.11	0.0069 1.18 / 0.24	0.0115 1.09 / 0.28
EuroStoxx 50	0.0052 0.60 / 0.55	0.0072 0.62 / 0.53	0.0052 0.60 / 0.55	0.0139 1.15 / 0.25	-0.0033 -0.13 / 0.90
DAX 30	0.0073 1.07 / 0.29	0.0140 1.53 / 0.13	0.0073 1.07 / 0.29	0.0140 1.47 / 0.14	0.0099 0.59 / 0.56
CAC 40	0.0090 1.04 / 0.30	0.0031 0.27 / 0.79	0.0090 1.04 / 0.30	0.0060 0.51 / 0.61	0.0029 0.13 / 0.90
FTSE 100	0.0031 0.46 / 0.65	0.0084 1.01 / 0.32	0.0031 0.46 / 0.64	0.0158 1.84 / 0.07	-0.0086 -0.58 / 0.56
MSCI EM	0.0084 0.78 / 0.44	0.0044 0.27 / 0.79	0.0084 0.75 / 0.45	0.0305 1.91 / 0.06	-0.0188 -0.57 / 0.57
Panel C: Period III (01/2003 – 12/2012)					
S&P 500	0.0127 2.05 / 0.04	-0.0016 -0.18 / 0.85	0.0127 2.05 / 0.04	-0.0016 -0.18 / 0.86	0.0023 0.12 / 0.90
EuroStoxx 50	0.0126 1.53 / 0.13	0.0025 0.22 / 0.82	0.0126 1.54 / 0.13	0.0065 0.56 / 0.58	-0.0001 -0.01 / 1.00
DAX 30	0.0137 1.65 / 0.10	0.0085 0.75 / 0.46	0.0137 1.65 / 0.10	0.0085 0.73 / 0.47	0.0100 0.40 / 0.69
CAC 40	0.0166 2.03 / 0.04	-0.0015 -0.14 / 0.89	0.0166 2.03 / 0.04	-0.0015 -0.13 / 0.90	-0.0046 -0.18 / 0.86
FTSE 100	0.0096 1.53 / 0.13	0.0028 0.35 / 0.73	0.0096 1.52 / 0.13	0.0110 1.31 / 0.19	-0.0151 -0.85 / 0.40
MSCI EM	0.0084 0.78 / 0.44	0.0044 0.27 / 0.79	0.0084 0.75 / 0.45	0.0305 1.91 / 0.06	-0.0188 -0.57 / 0.57

The table reports the results from median regressions (rather than OLS) applied to the standard dummy variables model. The two models are described in Eqs. (1) and (2) in Section 4. Rather than estimating these models using OLS, the table shows median regression coefficients to control for potential outliers (least-absolute-deviations regression). μ denotes the estimated intercept term, α_1 the coefficient that indicates the Halloween effect, and α_2 the coefficient on the dummy variable that controls for the January effect. All available index data are used in Panel A. Panel B accounts for the availability of suitable investment instruments, and Panel C captures the publication date of Bouman and Jacobsen‘s (2002) study. The first line reports the estimated coefficient. The second line contains the t-value (left) and the corresponding p-value (right).

Exhibit 5: Bootstrap simulation environment

<u>Bootstrap 1:</u> Strategy simulation



<u>Bootstrap 2:</u> Statistical inference

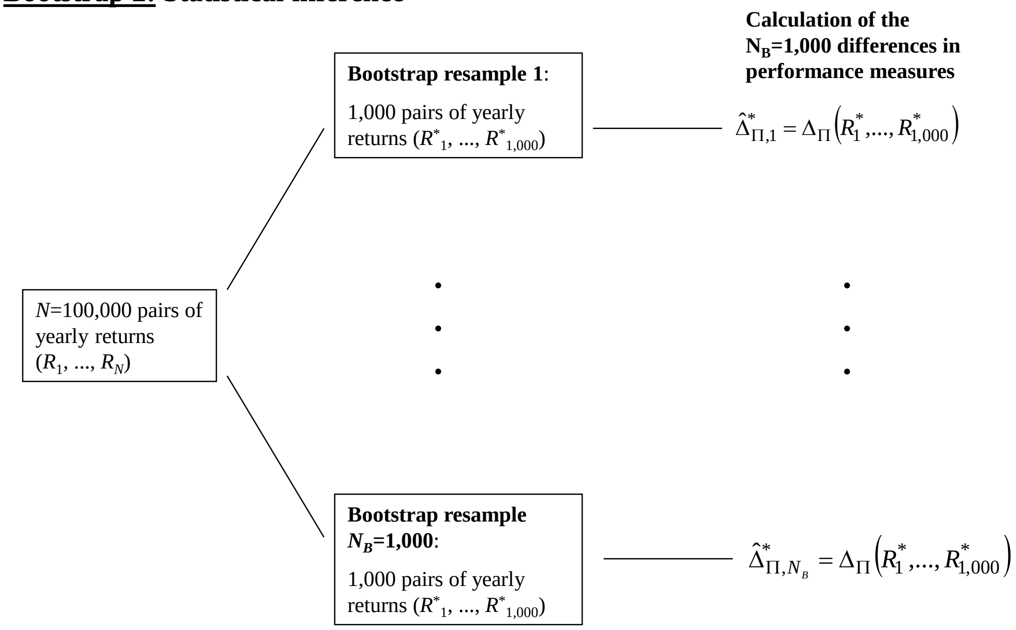


Exhibit 6: Baseline bootstrap simulation results

	Strategy	Mean excess return p.a.	Volatility	Sharpe ratio
Panel A: Period I (availability of stock market index data – 12/2012)				
S&P 500	Halloween Buy-and-hold Δ / Significance	5.26 7.30 -2.04 / ***	10.49 16.25 -5.76 / ***	0.501 0.449 0.052 / *
EuroStoxx 50	Halloween Buy-and-hold Δ / Significance	7.15 6.40 0.75 / -	13.16 20.31 -7.15 / ***	0.543 0.315 0.228 / ***
DAX 30	Halloween Buy-and-hold Δ / Significance	8.49 6.84 1.65 / ***	15.27 23.52 -8.25 / ***	0.556 0.291 0.265 / ***
CAC 40	Halloween Buy-and-hold Δ / Significance	6.52 6.10 0.42 / -	14.86 22.05 -7.19 / ***	0.439 0.277 0.162 / ***
FTSE 100	Halloween Buy-and-hold Δ / Significance	5.03 4.57 0.46 / -	10.71 17.38 -6.67 / ***	0.470 0.263 0.207 / ***
MSCI EM	Halloween Buy-and-hold Δ / Significance	12.28 12.23 0.05 / -	17.48 27.61 -10.13 / ***	0.703 0.443 0.26 / ***
Panel B: Period II (availability of stock market funds – 12/2012)				
S&P 500	Halloween Buy-and-hold Δ / Significance	5.26 7.30 -2.04 / ***	10.49 16.25 -5.76 / ***	0.501 0.449 0.052 / *
EuroStoxx 50	Halloween Buy-and-hold Δ / Significance	0.49 -2.56 3.05 / ***	13.30 20.06 -6.76 / ***	0.037 -
DAX 30	Halloween Buy-and-hold Δ / Significance	8.44 7.88 0.56 / -	15.66 24.36 -8.70 / ***	0.539 0.323 0.216 / ***
CAC 40	Halloween Buy-and-hold Δ / Significance	1.22 -1.28 2.50 / ***	12.69 19.47 -6.78 / ***	0.096 -
FTSE 100	Halloween Buy-and-hold Δ / Significance	2.19 0.85 1.34 / ***	9.44 15.33 -5.89 / ***	0.232 0.055 0.177 / ***
MSCI EM	Halloween Buy-and-hold Δ / Significance	11.14 14.99 -3.85 / ***	16.70 28.70 -12.00 / ***	0.667 0.522 0.145 / ***
Panel C: Period III (01/2003 – 12/2012)				
S&P 500	Halloween Buy-and-hold Δ / Significance	4.32 6.59 -2.27 / ***	9.87 15.87 -6.00 / ***	0.438 0.415 0.023 / -
EuroStoxx 50	Halloween Buy-and-hold Δ / Significance	1.66 3.61 -1.95 / ***	13.06 18.96 -5.90 / ***	0.127 0.190 -0.063 / ***
DAX 30	Halloween Buy-and-hold Δ / Significance	6.64 10.21 -3.57 / ***	15.54 22.58 -7.04 / ***	0.427 0.452 -0.025 / -

CAC 40	Halloween Buy-and-hold Δ / Significance	2.25 4.80 -2.55 / ***	12.26 18.44 -6.18 / ***	0.184 0.260 -0.076 / ***
FTSE 100	Halloween Buy-and-hold Δ / Significance	3.32 5.83 -2.51 / ***	9.41 15.01 -5.60 / ***	0.353 0.388 -0.035 / -
MSCI EM	Halloween Buy-and-hold Δ / Significance	11.14 14.99 -3.85 / ***	16.70 28.70 -12.00 / ***	0.667 0.522 0.145 / ***

The table reports the results from the baseline bootstrap simulations and shows the mean annual excess return, the volatility of excess returns, and the Sharpe ratio of the simulated Halloween strategy and the buy-and-hold benchmark. All available index data are used in Panel A. Panel B accounts for the availability of suitable investment instruments, and Panel C captures the publication date of Bouman and Jacobsen‘s (2002) study. “Δ” is the difference in a performance measure between the two simulated investment strategies. As described in Section 5.1, the results are based on 100,000 simulation runs in the first-step bootstrap for the simulation of the two strategies. The hypothesis tests in the second-step are based on Efron’s (1979) standard bootstrap method (with 1,000 drawings, each consisting of 1,000 elements). An illustration of the bootstrap simulation environment is shown in Exhibit 5. The shift from the stock market into the cash market in May, and the reverse shift in October under the Halloween strategy are executed with transaction costs of 50 bps. ***, **, and * denotes statistical significance at the 1%, 5%, and 10% level.

Exhibit 7: Modified bootstrap simulation results

	Strategy	Mean excess return p.a.	Volatility	Sharpe ratio
Panel A: Period I (availability of stock market index data – 12/2012)				
S&P 500	Halloween Buy&Hold Δ / Significance	5.16 7.12 -1.96 / ***	10.34 15.97 -5.63 / ***	0.499 0.446 0.053 / ***
EuroStoxx 50	Halloween Buy&Hold Δ / Significance	7.00 6.11 0.89 / *	12.96 19.84 -6.88 / ***	0.540 0.308 0.232 / ***
DAX 30	Halloween Buy&Hold Δ / Significance	8.43 6.65 1.78 / ***	14.95 22.83 -7.88 / ***	0.564 0.291 0.273 / ***
CAC 40	Halloween Buy&Hold Δ / Significance	6.56 6.03 0.53 / -	14.69 21.61 -6.92 / ***	0.447 0.279 0.168 / ***
FTSE 100	Halloween Buy&Hold Δ / Significance	5.03 4.56 0.47 / -	10.43 17.11 -6.68 / ***	0.482 0.267 0.215 / ***
MSCI EM	Halloween Buy&Hold Δ / Significance	12.28 12.42 -0.14 / -	17.12 27.01 -9.89 / ***	0.717 0.460 0.257 / ***
Panel B: Period II (availability of stock market funds – 12/2012)				
S&P 500	Halloween Buy&Hold Δ / Significance	5.16 7.12 -1.96 / ***	10.34 15.97 -5.63 / ***	0.499 0.446 0.053 / ***
EuroStoxx 50	Halloween Buy&Hold Δ / Significance	0.36 -2.78 3.14 / ***	12.63 19.19 -6.56 / ***	0.029 -
DAX 30	Halloween Buy&Hold Δ / Significance	8.28 7.46 0.82 / -	15.26 23.50 -8.24 / ***	0.543 0.317 0.226 / ***
CAC 40	Halloween Buy&Hold Δ / Significance	1.12 -1.41 2.53 / ***	12.05 18.73 -6.68 / ***	0.093 -
FTSE 100	Halloween Buy&Hold Δ / Significance	2.10 0.76 1.34 / ***	8.54 14.48 -5.94 / ***	0.246 0.052 0.194 / ***
MSCI EM	Halloween Buy&Hold Δ / Significance	11.10 14.75 -3.65 / ***	16.24 27.85 -11.61 / ***	0.683 0.530 0.153 / ***
Panel C: Period III (01/2003 – 12/2012)				
S&P 500	Halloween Buy&Hold Δ / Significance	4.36 6.62 -2.26 / ***	9.26 15.38 -6.12 / ***	0.471 0.430 0.041 / -
EuroStoxx 50	Halloween Buy&Hold Δ / Significance	1.52 3.52 -2.00 / ***	12.25 18.29 -6.04 / ***	0.124 0.192 -0.68 / ***
DAX 30	Halloween Buy&Hold Δ / Significance	6.49 10.10 -3.61 / ***	14.08 21.21 -7.13 / ***	0.461 0.476 -0.015 / -

CAC 40	Halloween Buy&Hold Δ / Significance	2.26 4.77 -2.51 / ***	11.65 17.89 -6.24 / ***	0.194 0.267 -0.073 / ***
FTSE 100	Halloween Buy&Hold Δ / Significance	3.24 5.71 -2.47 / ***	8.10 14.07 -5.97 / ***	0.400 0.406 -0.006 / -
MSCI EM	Halloween Buy&Hold Δ / Significance	11.10 14.75 -3.65 / ***	16.24 27.85 -11.61 / ***	0.683 0.530 0.153 / ***

The table reports the results from the modified bootstrap simulations and shows the mean annual excess return, the volatility of excess returns, and the Sharpe ratio of the simulated Halloween strategy and the buy-and-hold benchmark. In comparison with our original bootstrap design in Exhibit 6, this alternative guarantees that the monthly structure of any given year is maintained. All available index data are used in Panel A. Panel B accounts for the availability of suitable investment instruments, and Panel C captures the publication date of Bouman and Jacobsen‘s (2002) study. "Δ" denotes the difference in a performance measure between the two simulated investment strategies. As described in Section 5.1, the results are based on 100,000 simulation runs in the first-step bootstrap for the simulation of the two strategies. The hypothesis tests in the second-step are based on Efron's (1979) standard bootstrap method (with 1,000 drawings, each consisting of 1,000 elements). An illustration of the bootstrap simulation environment is shown in Exhibit 5. The shift from the stock market into the cash market in May, and the reverse shift in October under the Halloween strategy are executed with transaction costs of 50 bps. ***, **, and * denotes statistical significance at the 1%, 5%, and 10% level.

Exhibit 8: Bootstrap simulation results with different transaction costs

	Strategy	Mean excess return p.a.	Volatility	Sharpe ratio
Panel A: Transaction costs of 0.1%				
S&P 500	Halloween Buy&Hold Δ / Significance	5.15 6.51 -1.36 / ***	9.32 15.35 -6.03 / ***	0.553 0.424 0.129 / ***
EuroStoxx 50	Halloween Buy&Hold Δ / Significance	2.42 3.61 -1.19 / ***	12.33 18.32 -5.99 / ***	0.196 0.197 -0.001 / -
DAX 30	Halloween Buy&Hold Δ / Significance	7.37 10.07 -2.70 / ***	14.16 21.14 -6.98 / ***	0.520 0.476 0.044 / -
CAC 40	Halloween Buy&Hold Δ / Significance	3.10 4.86 -1.76 / ***	11.76 17.90 -6.14 / ***	0.264 0.272 -0.008 / -
FTSE 100	Halloween Buy&Hold Δ / Significance	4.10 5.77 -1.67 / ***	8.16 14.04 -5.88 / ***	0.502 0.411 0.091 / ***
MSCI EM	Halloween Buy&Hold Δ / Significance	11.95 14.68 -2.73 / ***	16.35 27.81 -11.46 / ***	0.731 0.528 0.203 / ***
Panel B: Transaction costs of 1.0%				
S&P 500	Halloween Buy&Hold Δ / Significance	3.24 6.60 -3.36 / ***	9.16 15.38 -6.22 / ***	0.354 0.429 -0.075 / **
EuroStoxx 50	Halloween Buy&Hold Δ / Significance	0.49 3.52 -3.03 / ***	12.14 18.27 -6.13 / ***	0.040 0.193 -0.153 / ***
DAX 30	Halloween Buy&Hold Δ / Significance	5.37 10.11 -4.74 / ***	13.93 21.17 -7.24 / ***	0.385 0.478 -0.093 / ***
CAC 40	Halloween Buy&Hold Δ / Significance	1.29 4.92 -3.63 / ***	11.55 17.91 -6.36 / ***	0.112 0.275 -0.163 / ***
FTSE 100	Halloween Buy&Hold Δ / Significance	2.14 5.75 -3.61 / ***	7.98 14.05 -6.07 / ***	0.268 0.409 -0.141 / ***
MSCI EM	Halloween Buy&Hold Δ / Significance	10.07 14.88 -4.81 / ***	16.07 27.94 -11.87 / ***	0.627 0.533 0.094 / ***

The table reports the results from bootstrap simulations and shows the mean annual excess return, the volatility of excess returns, and the Sharpe ratio of the simulated Halloween strategy and the buy-and-hold benchmark. In comparison with our modified bootstrap design in Exhibit 7, this alternative uses different transaction costs. In particular, the shift from the stock market into the cash market in May, and the reverse shift in October under the Halloween strategy are executed with transaction costs of 0.1% (Panel A) or 1.0% (Panel B). The sample period is 01/2003-12/2012 (latest subsample after publication of Bouman and Jacobsen‘s (2002) study). “Δ” denotes the difference in a performance measure between the two simulated investment strategies. As described in Section 5.1, the results are based on 100,000 simulation runs in the first-step bootstrap for the simulation of the two strategies. The hypothesis tests in the second-step are based on Efron’s (1979) standard bootstrap method (with 1,000 drawings, each consisting of 1,000 elements). An illustration of the bootstrap simulation environment is shown in Exhibit 5. ***, **, and * denotes statistical significance at the 1%, 5%, and 10% level.

Exhibit 9: Bootstrap simulation results with leverage

	Strategy	Mean excess return p.a.	Volatility	Sharpe ratio
Panel A: Transaction costs of 0.5%				
S&P 500	Halloween Buy&Hold Δ / Significance	8.64 6.54 2.10 / ***	19.31 15.40 3.91 / ***	0.447 0.425 0.022 / -
EuroStoxx 50	Halloween Buy&Hold Δ / Significance	2.93 3.56 -0.63 / -	24.94 18.31 6.63 / ***	0.117 0.194 -0.077 / ***
DAX 30	Halloween Buy&Hold Δ / Significance	12.75 9.98 2.77 / ***	29.63 21.11 8.52 / ***	0.430 0.472 -0.042 / *
CAC 40	Halloween Buy&Hold Δ / Significance	4.52 4.83 -0.31 / -	23.78 17.89 5.89 / ***	0.190 0.270 -0.08 / ***
FTSE 100	Halloween Buy&Hold Δ / Significance	6.25 5.74 0.51 / ***	16.71 14.08 2.63 / ***	0.374 0.408 -0.034 / -
MSCI EM	Halloween Buy&Hold Δ / Significance	23.01 14.89 8.12 / ***	35.72 27.93 7.79 / ***	0.644 0.533 0.111 / ***
Panel B: Transaction costs of 0.1%				
S&P 500	Halloween Buy&Hold Δ / Significance	10.30 6.45 3.85 / ***	19.54 15.30 4.24 / ***	0.527 0.422 0.105 / ***
EuroStoxx 50	Halloween Buy&Hold Δ / Significance	4.72 3.58 1.14 / *	25.32 18.33 6.99 / ***	0.186 0.195 -0.009 / -
DAX 30	Halloween Buy&Hold Δ / Significance	14.71 10.09 4.62 / ***	30.23 21.15 9.08 / ***	0.485 0.477 0.008 / -
CAC 40	Halloween Buy&Hold Δ / Significance	6.23 4.88 1.35 / **	24.27 17.96 6.31 / ***	0.257 0.272 -0.015 / -
FTSE 100	Halloween Buy&Hold Δ / Significance	7.98 5.65 2.33 / ***	16.96 14.07 2.89 / ***	0.471 0.402 0.069 / **
MSCI EM	Halloween Buy&Hold Δ / Significance	25.14 15.01 10.13 / ***	36.51 27.98 8.53 / ***	0.689 0.536 0.153 / ***

The table reports the results from bootstrap simulations with leverage and shows the mean annual excess return, the volatility of excess returns, and the Sharpe ratio of the simulated Halloween strategy and the buy-and-hold benchmark. In comparison with our modified bootstrap design in Exhibit 7, this alternative holds no stocks from May through October and a leveraged position of 200% stocks is accumulated from November through April. The sample period is 01/2003-12/2012 (latest subsample after publication of Bouman and Jacobsen‘s (2002) study). “Δ” denotes the difference in a performance measure between the two simulated investment strategies. As described in Section 5.1, the results are based on 100,000 simulation runs in the first-step bootstrap for the simulation of the two strategies. The hypothesis tests in the second-step are based on Efron’s (1979) standard bootstrap method (with 1,000 drawings, each consisting of 1,000 elements). An illustration of the bootstrap simulation environment is shown in Exhibit 5. The shift from the stock market into the cash market in May, and the reverse shift in October under the Halloween strategy are executed with transaction costs of 0.5% (Panel A) and 0.1% (Panel B). ***, **, and * denotes statistical significance at the 1%, 5%, and 10% level.